DSP Exercise Set 1

1.1a The energy of a signal s is E. What is the energy of $\hat{z}^{-1}s$? Of gs (g is a gain)? What can be said about the energy of the sum of two signals? (Note that the energy of the sum of two signals can be zero! When can this happen?)

1.1b Thomas Alva Edison didn't believe that AC electricity could be useful, since the current first went one way and then returned, producing no net effect. Why was Edison wrong?

1.2a The *power* of an analog signal is defined as the energy of the signal over some time interval divided by duration of that interval. What is the power of the analog sinusoid $s(t) = A\sin(\omega t)$ over any whole number of cycles?

1.2b The square root of a signal's energy defines a kind of average signal value, called the **R**oot **M**ean **S**quared (RMS) value. What is the RMS of an analog sinusoid?

1.2c A signal's *peak factor* is defined to be the ratio between its highest value and its RMS value. What is the peak factor of the analog sinusoid? The sum of N sinusoids of different frequencies?

1.3 An *eigensignal* of an operator is a signal that is unchanged by the operator except for a gain (if the operator is $\hat{O}$ then s is an eigensignal with eigenvalue λ if $\hat{O}s = \lambda s$). Show that the exponential signal $s_n = Ae^{\Lambda n}$ is an *eigensignal* of the time *advance* operator. What is its eigenvalue? What about the time *delay* operator?

1.4 The real digital sinusoid $s_n = A\sin(\omega n + \phi)$ is the eigensignal of an operator that contains $\hat{z}^{-1}$ and $\hat{z}^{-2}$. Can you find this operator?

1.5a The sum of an analog sinusoid of period 4 seconds and one of period 6 seconds is periodic with period 12 seconds. This is because the first sinusoid completes 3 full periods and the second 2 complete periods in this time. Is the sum of 2 analog sinusoids always periodic? Explain.

1.5b When is the sum of two analog sinusoids a sinusoid? What happens when their frequencies are very close? Prove your assertion using trigonometric identities.

1.6 We saw that the DC signal $s_n = \cdots 1, 1, 1, 1, \cdots$ has only frequency $f = 0$, and that the Nyquist signal $s_n = \cdots 1, -1, 1, -1, \cdots$ has only frequency $f = \frac{1}{2}$ ($\omega = \pi$). Without computing a DFT, what frequencies are in the spectrum of the signal that alternates between 0 and 1 ($s_n = \cdots 0, 1, 0, 1, \cdots$)? What frequency is in the signal $s_n = \cdots 0, 1, 0, -1, 0, 1, 0, -1, \cdots$?

1.7 The purpose of this question is to understand why the digital frequency is the analog frequency divided by the sampling rate. Plot one second of an analog sinusoid $s(t) = \sin(2\pi f t)$ with frequency $f = 10$ Hz by computing it every millisecond and connecting the samples. Now *sample* it at a rate of 50 Hz by superposing a circle on the curve at the sample times. Repeat for a sinusoid of frequency $f = 1$ Hz with the same millisecond resolution but plot for 10 seconds. Now sample at rate 5 Hz. Explain what you observed.